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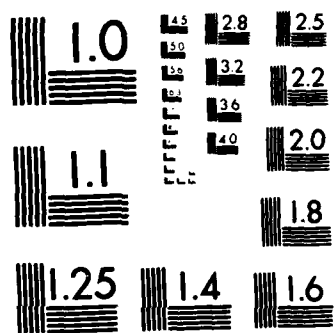
THE PLASMA ASSISTED MODIFIED BETATRON(U) NAVAL RESEARCH 1/1
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THE PLASMA ASSISTED MODIFIED BETATRON

I. Introduction

In order to obtain a high current, high energy electron beam, several laboratories are giving serious consideration to the modified betatron.¹⁻³ The idea is that the net space charge forces, which is outward in the poloidal plane, can be confined with a strong toroidal field. Theory and particle simulations³ show that for at least one poloidal drift orbit, much higher currents can be confined in a modified betatron than in a conventional betatron. Self-consistent fluid formulations^{4,5} have shown that cold fluid equilibriums exist and can be accelerated. Recently, the theory was extended to include transverse pressure.⁶

However, while the modified betatron offers many advantages, it is possible, but not certain, that future experiments may face several difficulties. First of all, one must inject the beam across toroidal field lines in order to have a beam centered in the liner. The current scheme⁷ proposes to shoot the beam into the toroidal vacuum chamber near the liner. The drift due to the focusing fields and image fields cause the beam to drift in the poloidal plane around the liner. In one toroidal transit (about 20 nsec) it should drift enough to miss the injector. In one poloidal drift time (several hundred nanoseconds), external fields can be changed to bring the beam slightly in from the liner so that it misses the injector again and henceforth. On a longer time scale, wall resistivity causes the beam to drift inward.

However there is a significant range of beam currents for which wall resistivity causes the beam to drift outward if it is near the liner, but inward if it is near the center. Since it is unlikely that the beam can reverse drift directions on the way in, the injection scheme of Ref. 7 appears to be viable only for fairly low beam currents.

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A second possible difficulty concerning the modified betatron is the diamagnetic to paramagnetic transition.^{4,5} Depending on whether the net self force is outward or inward in the poloidal plane, the net electron drift velocity is in the diamagnetic or paramagnetic direction. As the high current beam accelerates, it makes a transition from diamagnetic to paramagnetic current flow. It has been shown that subject only to the constraint that the acceleration time $\tau_a \sim 10^{-3}$ sec is very long compared to the drift time, $\tau_D \gtrsim 10^{-7}$ sec, this transition must suddenly change the topology of the beam orbits in the poloidal plane.^{4,5} Whether the beam can survive such a sudden, violent perturbation is an open question.

Finally, although the focusing fields in the modified betatron stabilize the $\ell = 1$ resistive wall instability, $\ell = 2$ modes are still unstable and pose a real threat to beam confinement in the modified betatron.⁸ These difficulties are discussed in more detail in Sec. II.

The root cause of all of these problems is that the beam self fields in the modified betatron are outward in the poloidal plane. By making the self fields inward, all three of these difficulties can be alleviated. One way of doing this is with a very low density background plasma. This is discussed in Sec. III.

Section IV discusses other physics issues concerning the plasma assisted modified betatron, among them plasma production, plasma response to the injected beam and ion resonance and streaming instabilities.

II. Injection, Diamagnetic Transition and $\ell = 2$ Resistive Wall Instability in a Vacuum Modified Betatron

This section discusses what appears to be three fundamental issues regarding the high current modified betatron operating with a vacuum background: beam injection, the diamagnetic to paramagnetic transition, and the $\ell = 2$ resistive wall instability.

A. Injection

One of the important issues for the modified betatron is injecting the beam. The present thinking for the NRL modified betatron experiment is described in Kapetanacos, et al.⁷ The beam is injected near the liner and drifts around the edge of the liner through a combination of drift generated by the focusing fields (field index) and image forces. The former is directed inward in the poloidal plane, the latter, outward. At high beam current the latter dominates and the beam rotates in say a counterclockwise direction as in Fig. 3 of Ref. 7 for the case of a ten kilo Amp beam. If the combination of forces is large enough, the drift velocity will be great enough so that after one toroidal revolution, the beam will be displaced in the poloidal plane by a large enough distance that it misses the injector. Then, since it has many more toroidal transits before it would hit the injector again, macroscopic fields could change sufficiently to bring the beam into the center.

One potential problem with this scheme, which is not addressed in Ref. 7 is that for high current beams, the net poloidal force on the beam is outward near the liner, but inward when the beam is at the center. Thus, as the beam continues to spiral in the poloidal plane, at some point it must reverse direction. To see this more quantitatively, if the field index of the beam is $\frac{1}{2}$, the focusing field produces an

inward poloidal force on a charge q of $q \frac{B_z}{2} \frac{\rho}{R_0}$ where R_0 is the major radius of the equilibrium orbit, B_z is the vertical field and ρ is the displacement of the beam from the equilibrium orbit in the poloidal plane $\rho^2 = (R - R_0)^2 + z^2$. The image electric force for a cylindrical system is given by

$$E_{im} \text{ (cgs)} = \frac{2I(\text{Amps}) \rho(\text{cm})}{10(a^2 - \rho^2)} \quad (1)$$

where a is the minor radius of the liner.

The actual image force is canceled in part by magnetic forces and also by any fractional neutralization f . If the beam is near the wall ($\rho \approx a$) the net poloidal drift velocity is given by

$$\frac{V_D}{C} = [(\gamma^{-2} - f) \frac{I(\text{Amps})}{10\delta(\text{cm}) B_\theta(\text{Gauss})} - \frac{B_z a}{2B_\theta R_0}] \quad (2)$$

where δ is the distance from the beam center to the liner. Since the beam enters the toroidal liner right near the outer edge of the liner,^{7,9} δ is roughly equal to the beam radius ρ_b .

If the beam is near the center (so $\rho \ll a$), the poloidal drift is given by

$$\frac{V_D}{C} = \frac{2I(\text{Amps}) \rho(\text{cm})}{10a^2(\text{cm}) B_\theta(\text{Gauss})} \left(\frac{1}{\gamma^2} - f \right) - \frac{B_z \rho}{2B_\theta R_0} \quad (3)$$

For the case of a vacuum modified betatron, the current can be classified as being in one of three ranges:

I. High Current

$$5 a(\text{cm}) \gamma^2 B_\theta (\text{Gauss}) \left(\frac{B_z a}{2B_\theta R_o} \right) < I(\text{A}) \quad (4)$$

II. Intermediate Current

$$10 \delta(\text{cm}) B_\theta (\text{Gauss}) \gamma^2 \left(\frac{B_z a}{2B_\theta R_o} \right) < I(\text{A}) < 5 \gamma^2 a(\text{cm}) B_\theta (\text{Gauss}) \left(\frac{B_z a}{2B_\theta R_o} \right) \quad (5)$$

III. Low Current

$$I(\text{A}) < 10 \gamma^2 \delta(\text{cm}) B_\theta (\text{Gauss}) \left(\frac{B_z a}{2B_\theta R_o} \right). \quad (6)$$

In the high current regime, the forces on the beam in the poloidal plane are outward and the beam always rotates in the counterclockwise direction. In the intermediate regime, the forces are outward when the beam is near the wall, but inward when the beam is near the center. Thus in this current regime, the beam must reverse its direction of rotation before it gets to the center. Also, at some radius between the center and the wall, the beam will have zero poloidal drift velocity. It seems likely that some time after injection, an intermediate current beam will stagnate around this point and gradually fill the chamber. In the low current regime, the inward focusing forces always dominate and the beam rotation is clockwise.

It is also worth noting that if the liner is resistive, the beam will spiral inward if the net force is inward and visa versa. Thus a resistive wall can only trap a class III low current beam. It is possible that an intermediate current beam can be trapped if it can be brought sufficiently near the center that the net forces are inward. For the

parameters of the NRL modified betatron experiment,^{2,3} $B_0 = 2 \times 10^3$, $\gamma = 4$, $R_0 = 10^2$, $a = 15$, $\delta = 2$, $B_z = 140$, the lower and upper currents of the intermediate range are 3.2×10^3 A and 1.2×10^4 A. Thus the maximum current which can be trapped by wall resistivity in a vacuum modified betatron is about 3 kA. Actually however, the maximum current is much less because at the 3.2 kA level the poloidal drift is zero, so the beam will strike the injector after one toroidal transit. Note that the model here assumed a cylindrical system. It has recently been shown¹⁰ that toroidal effects can complicate this picture somewhat by allowing more complex banana shaped orbits. The authors of Ref. 10 suggest an injection scheme where the beam orbit is shifted from a banana to circular orbit by a pulsed change in toroidal field.

B. Diamagnetic to Paramagnetic Transition

Let us imagine that the beam has been injected and is centered in the modified betatron. The question then is the individual particle orbits in the beam. Each particle feels an inward force due to the focusing fields and an outward force due to the self fields. If the latter dominates, the particle has an $F \times B$ drift in the counterclockwise direction, analogous to the counterclockwise whole beam drift for an outward image force discussed in the previous section. Then the J (poloidal) B (toroidal) force is inward. In this case, the beam is said to be diamagnetic. This is analogous to the terminology in plasma physics, where the poloidal current is diamagnetic if it gives an inward force. On the other hand, if the focusing force dominates, the $F \times B$ drift is clockwise and the beam is said to be paramagnetic.

If the beam has uniform density and radius ρ_b , the outward force is the electrostatic force canceled by the magnetic force and fractional charge neutralization. A test charge q at $\rho = \rho_b$ feels an outward force

$$F_s(\text{esu}) = q(\gamma^{-2} - f) \frac{I(\text{AMPS})}{10 \rho_b(\text{cm})}. \quad (7)$$

The inward focusing force is given by

$$F_f = -q \frac{B_z}{2} \frac{\rho_b}{R} \quad (8)$$

so the condition for a paramagnetic beam is for a vacuum modified betatron is

$$\gamma^{-2} \frac{I(\text{AMPS})}{10 \rho_b(\text{cm})} < \frac{B_z(\text{Gauss})}{2} \frac{\rho_b}{R}. \quad (9)$$

Note that a high current beam is generally diamagnetic. However as it accelerates, the left hand side becomes smaller as γ increases, and the right hand side becomes larger because B_z is proportional to γ . Thus for a high current beam which starts out diamagnetic, as it accelerates it ultimately makes a transition and becomes paramagnetic.

One might think this simply means that the poloidal rotation of the particle stops and changes direction. Actually the situation is considerably more complex, and also worse from the point of view of operation of the modified betatron. In a recent series of papers,⁴⁻⁶ it has been shown that subject only to the constraint that the acceleration time is very long compared to the drift time, an approximation well-satisfied in the NRL modified betatron (but not satisfied at all in particle simulations of the device), the diamagnetic to paramagnetic transition necessarily results in a change of topology of the beam. The outer beam particles first become paramagnetic and in doing so scrape off the edge of the beam and form a large minor radius hollow beamlet. As the energy continues to increase, the scrape-off point moves inside the beam and inner beam particles continue to add to

the outside of the hollow beamlet. The process is completed when the beam has turned itself completely inside out and has gone from a solid to a hollow beam.

Although this process is complicated, it is very easy to see that in making the transition, the beam must turn itself inside out. To show this, it is only necessary to invoke the conservation of toroidal canonical momentum P_θ . If the poloidal magnetic field is given by $\nabla\psi \times \hat{i}_\theta / R$, then

$$P_\theta = \gamma m R v_\theta + \frac{q\psi}{c}. \quad (10)$$

To evaluate P_θ , note that $\gamma = (E - q\phi)/mc^2$ where ϕ is the electrostatic potential. For a cylindrical beam of radius r_b ,

$$\phi(\rho) = \begin{cases} \pi n q \rho^2 & \rho < \rho_b \\ \pi b q r_b^2 (\ln(\rho/\rho_b) + 1) & \rho_b < \rho. \end{cases} \quad (11)$$

The flux ψ has three components. First there is the flux of the vertical field itself, assumed uniform

$$\psi_v = \frac{B_z R^2}{2}. \quad (12)$$

Secondly, there is the flux associated with the focusing field. If the field index is $\frac{1}{2}$, this is

$$\psi_f = \frac{B_z \rho^2}{4}. \quad (13)$$

Note that ψ_f has this form both for $\rho < \rho_b$ and $\rho > \rho_b$. Finally, there is the flux associated with the self field,

$$\psi_s = (V_A/c)^2 R\phi(\rho). \quad (14)$$

Thus if $V_\theta \approx c$, near the axis ($\rho \approx 0$) one has that

$$P_\theta = \frac{qB_z \rho^2}{4c} + \frac{nR_o q^2 \pi}{\gamma^2 c} \rho^2 + P_\theta(\rho = 0). \quad (15)$$

Since $qB_z < 0$ for the modified betatron, one has the result that if n is large enough that the second term dominates (that is, if the beam is diamagnetic), $P_\theta(\rho = 0)$ is a relative minimum. However far from the beam P_θ is dominated by the focusing forces which have the opposite sign. Thus P_θ as a function of ρ for a diamagnetic beam is as shown in Fig. 1a. On the other hand, if the beam is paramagnetic, the first term on the right hand side of Eq. (15) dominates so $P_\theta(\rho = 0)$ is a relative maximum, and $P_\theta(\rho)$ is shown in Fig. 1b.

The crucial point now is that in a configuration which has θ symmetry, P_θ is an exact constant of motion. Consider then the orbits at $\rho = 0$ and $\rho = \rho_b$ for a diamagnetic beam. The former is inside the latter and has a lower value of P_θ according to Fig. 1a. After diamagnetic to paramagnetic transition, these values of P_θ cannot change. However a paramagnetic beam has the reference orbit at a relative maximum so that this must correspond to the orbit initially at $\rho = \rho_b$ in the diamagnetic beam. Thus in making transition the beam must, at the very least, turn itself inside out.

Actually, as shown in Refs. 4-6, not only does the beam turn itself inside out, it transitions from a solid to hollow beam. In doing so, the beam

could strike the wall and thereby disrupt. Conditions for the beam to remain confined on transition are given in Refs. 4-6. However even if the beam does remain initially confined, it is not certain it can remain confined long after suffering such a violent perturbation. If nothing else, the hollow profile produced is diocotron unstable.

Finally, we point out that particle simulations of the beam passing through transition have been done. These are discussed in the Appendix.

C. Resistive Wall Instability

One other potential difficulty with the modified betatron is the resistive wall instability. If a beam of density n and radius ρ_b centered in a cylindrical tube of radius a , the frequency of a perturbation at frequency varying like $\exp i\ell\phi$ is⁸

$$\omega = [(\ell - 1) + \left(\frac{\rho_b}{a}\right)^{2\ell}] \frac{V_\phi}{\rho} - \omega_b \quad (16)$$

where V_ϕ is the rotation frequency of the electrons

$$V_\phi = \frac{-c}{B_\theta} (\gamma^2 - f) 2\pi n q \rho \quad (17)$$

and ω_b is the frequency of rotation generated by the focusing fields,

$$\omega_b = - \frac{c B_z}{2 B_\theta R_o} \quad (18)$$

The focusing fields produce a rotation in the negative θ direction. Since $q < 0$, the rotation of the beam itself is in the positive θ direction for the case of a vacuum beam, $f = 0$. The sign of the frequency is such that as long

as $\omega > 0$, wall resistivity gives rise to growth of this mode. This can be understood by noting that since the beam has only negative charge, the net force from any perturbation must be outward. Thus, as mentioned in Sec. III A, wall resistivity will cause the beam to spiral outward, corresponding to instability. Since the natural frequency of the $l = 1$ mode is very low, the sign of this frequency can be changed by the focusing fields, thereby stabilizing this mode. The condition is that the beam current, as defined in Sec. II A be in the low or intermediate regime. However the $l = 2$ mode has a significantly larger frequency so that in the vacuum modified betatron, it cannot be stabilized by the focusing fields.

III. A Plasma Assisted Modified Betatron

In the previous section we have seen that the modified betatron is beset by three potentially very serious problems: the difficulty of injection at high or intermediate range currents, the problem of the diamagnetic to paramagnetic transition which occurs even well into the low current range, and the problem of $\ell \geq 2$ resistive wall instability. However all of these difficulties resulted from the fact that $\gamma^{-2} - f > 0$, or that the beam self forces are outward. One possible cure for all these problems then is to operate the high current modified betatron in the presence of a low density background plasma, so that $\gamma^{-2} - f$ changes sign. This changes the sign of the self forces in the poloidal plane from outward to inward and provides cohesion for the beam itself. On injection, the image forces are now toward the center so that wall resistivity can trap the beam in the center. The beam will always be paramagnetic, so the problems of making transition are eliminated. Also, the frequency of the diocotron mode will be negative so that the resistive wall instability will be stabilized. The plasma densities required are low. For instance for a 10 kA, 2 cm radius beam with $\gamma = 4$, background ion density of order 10^{10} cm^{-3} is required. For a 1 kA beam, it is an order of magnitude lower.

Although we envision the betatron as operating in the presence of a background plasma, the scheme proposed here has little in common with a "plasma betatron". There, the beam is formed from runaway electrons and the beam density is small compared to the background plasma density.¹¹⁻¹³ An alternate scheme involves injection of an astron gun produced electron ring into a high density collisional plasma.¹⁴ In the scheme proposed here, a beam is still externally injected, and the preformed plasma density is low compared to the beam density.

IV. Physics Issues Relating to the Plasma Assisted Modified Betatron

In this section we consider three physics issues relating to the proposed plasma assisted modified betatron. They are producing the low density fully ionized plasma in the toroidal system, the response of this plasma to the injected beam, and the ion resonance and streaming instability.

A. Producing the Background Plasma

Producing a fully ionized plasma at a density as low as 10^{10} cm^{-3} appears to present some experimental difficulties. The most likely approach would be to produce the plasma at much higher density and let it expand into the entire toroidal chamber. The fact that there is a vertical field which is typically five to ten percent of the toroidal field means that the interior of the chamber is accessible along a field line from the outside. One possible scheme would then be to distribute a large number of very small plasma guns along the top of the liner. These could be made to fire simultaneously so that plasma would line the top of the chamber. As it drifted in along the field the density would decrease due to the expansion. The plasma would expand along the field, (and also outward in major radius) filling the torus. Since the expansion velocity is about 10^6 cm/sec , it would take many microseconds for the torus to fill. However this is a very long time compared to the 20 nsec transit time of the beam in major radius. Thus when plasma conditions are optimum, the beam would be fired in. The best way to determine the optimum plasma configuration is almost certainly experimentally, with a small scale experimental program.

B. The Plasma Response to the Injected Beam

The system envisioned has the beam injected into a very low density

plasma in a modified betatron configuration. The question then is how does the plasma respond to the beam, and more specifically how is f related to plasma, beam and system parameters. We treat the electron and ion responses separately. Throughout, we assume that the background plasma has sufficiently small density compared to the beam, that the electric fields from the beam dominate those from the plasma. Since the plasma is nearly at rest, there is no γ^{-2} cancellation of self electric fields.

The plasma electrons react on two time scales, the fast inertial time scale and the slow collisional time scale. When the beam enters the plasma, a strong inward electric field is set up, $\underline{E} = -2\pi n e \rho \underline{i}_\rho$, where we have adopted a cylindrical model for the beam and ρ is the radius (in the poloidal plane). Assuming this field is set up slowly compared to an inverse cyclotron time (2.5×10^{-10} sec for a 2 KG field), the beam electrons respond by $\underline{E} \times \underline{B}$ drifting in the θ direction and drifting outward due to the inertial drift. It is the inertial drift

$$v_{Di} = \frac{d\rho}{dt} = \frac{-c}{B\omega_{ce}} \frac{dE}{dt} = \frac{-c}{B\omega_{ce}} \frac{d}{dt} 2\pi n_b q \rho \quad (19)$$

which expels the electrons, (note $q = -|e|$) from the beam. If an electron starts out at ρ_1 when $n_b = t = 0$, Eq. (19) can be integrated once to give

$$\rho = \frac{\rho_1}{1 - \frac{\omega_{be}^2}{2\omega_{ce}^2}}. \quad (20)$$

Equation (20) has an apparent divergence, but of course the expression for E_ρ and ρ are valid only for $\rho < \rho_b$, the beam radius. However, Eq. (20) does show that the electrons are totally expelled by the beam if $\frac{\omega_{be}^2}{2\omega_{ce}^2} > 1$. For our

10 kA modified betatron parameters, $\omega_{be}^2/2\omega_{ce}^2 \gtrsim 0.2$, so about 20% of the plasma electrons are expelled from the beam region as the fields are being set up.

Now consider the longer (collisional) time scale. Electron-ion collisions in the plasma cause a drag force which gives rise to an outward drift

$$v_{dv} = \frac{dp}{dt} = -\frac{v}{\omega_c} \frac{c}{B} E = \frac{v \rho \omega_{be}^2}{2\omega_{ce}^2}. \quad (21)$$

Thus the electron radius increases exponentially in time with growth rate $v \omega_{be}^2/2\omega_{ce}^2$. Classically

$$v = \frac{n_p \lambda}{3.4 \times 10^5 T_e^{3/2}}. \quad (22)$$

For temperatures of about 1 eV and $n_p \sim 10^{10} \text{ cm}^{-3}$, the Coulomb logarithm $\lambda \approx 10$, so $v \sim 3 \times 10^5$. Thus the remaining electrons are expelled on a time scale of about 20 μsec .

Since the electrons are forced away from the beam on a 20 μsec time scale, and even without the beam, the electrons cannot be confined in a toroidal chamber, we expect the electrons to be expelled on a time scale of some tens of microseconds. This time is long compared to the time for the beam to center itself, but short on the time scale of the beam acceleration. Thus, once the beam begins to accelerate, there should be virtually no plasma electrons present.

For the ions however, the story is different because there is a strong attractive force between the ions and the beam.

To continue, we consider the ion response. If an ion is trapped near the center of the beam, its oscillation frequency is $\omega_i = (2\pi n_b e^2/M)^{1/2} \sim$

$2 \times 10^8 \text{ sec}^{-1}$ for protons in our standard 10 kA beam. Since this is ten times larger than the ion cyclotron frequency, the ions are effectively unmagnetized. The ion oscillation time is also much less than the poloidal drift time of the beam.

Our model is the same as that in a recent study of the ion resonance instability,¹⁵ namely that the ion is initially at rest and then when the beam enters, the ion begins to oscillate due to the electric field. Since ions initially within the beam do not leave, and other ions initially outside the beam spend at least part of their oscillation inside the beam, the ion density in the beam increases. We will now estimate this increase.

To estimate the steady state ion density, we assume the system is cylindrically symmetric about the beam center. Furthermore all ions oscillate with slightly different frequencies so that after several oscillations, the ions phase mix and are distributed uniformly along their phase orbits. Then, it has been shown that the ion distribution function in velocity space is

$$f_1(H, L) = \frac{2n_1 \delta(L) \omega(H) M^2}{\pi e |\partial \phi / \partial \rho|_{\rho = \rho(H)}} \quad (23)$$

where the constants of motion are

$$H = 1/2 M V^2 + q \phi(\rho) \quad (a)$$

(24)

$$L = m v \rho \quad (b)$$

$\rho(H)$ is the maximum radius of an ion with energy H , $\omega(H)$ is the oscillation frequency of an ion with energy H , $\phi(\rho)$ is the electrostatic potential of the beam and n_1 is the preformed plasma ion density. If the ion orbit is entirely

within the beam

$$\left| \frac{\partial \phi}{\partial \rho} \right| = 2\pi n_b e \rho \quad (a)$$

$$\rho(H) = (2H/M\omega_i^2)^{1/2} \quad (b) \quad (25)$$

$$\omega(H) = \omega_i. \quad (c)$$

If the ion is initially outside the beam

$$\left| \frac{\partial \phi}{\partial \rho} \right| = 2\pi n_b e \rho_b^2 / \rho \quad (a) \quad (26)$$

$$\rho(H) = \rho_b \exp \left[\frac{H}{H(\rho_b)} - 1 \right] \quad (b)$$

where $H(\rho_b) = q \phi(\rho_b)$. The frequency of the ion outside the beam cannot be computed in closed form. As an approximation to it, use the frequency of an ion which rotates around the beam

$$\omega(H) = \omega_i \rho_b / \rho(H). \quad (27)$$

If $\rho(H) \gg \rho_b$, it is not difficult to see that this estimate is off by a factor of order $[\ln \rho(H)/\rho_b]^{1/2}$.

The total number of ions trapped in the beam is then

$$N_1 = 2\pi \int_0^{\rho_b} \rho d\rho \left\{ \int_{e\phi_I}^{H(\rho_b)} dH + \int_{H(\rho_b)}^H dH \frac{2n_1 \rho(H) \omega(H) (M/2)^{1/2}}{\pi e \left| \frac{\partial \phi}{\partial \rho} \right| \rho(H) \rho(H - e\phi(\rho))^{1/2}} \right\} \quad (28)$$

The first integral, labeled I is just $\pi \rho_b^2 n_1$ since it represents those ions originally in the beam. The last integral II, we only estimate. To do so,

set

$$\frac{\rho(H) \omega(H)}{e \left| \frac{\partial \phi}{\partial \rho} \right| \rho(H)} = \frac{\rho(H)}{M \omega_1 \rho_b} \quad (a)$$

$$r(H) \sim d \quad (b) \quad (29)$$

$$(H - e\phi)^{1/2} \sim (H(d))^{1/2} \quad (c)$$

where d is the maximum radius in the poloidal plane. In this case, the integral $II \sim \pi n_1 \rho_b d (1 + \ln d/\rho_b)^{1/2}$ so that

$$N = n_1 \pi \rho_b^2 \left(1 + \frac{d}{\rho_b} (1 + \ln d/\rho_b)^{1/2}\right). \quad (30)$$

Thus on an ion oscillation time scale, the ion density in the beam is significantly enhanced. If the beam is centered as it is in steady state so $d/\rho_b \sim 5$, the ion density might be enhanced by nearly an order of magnitude. If the beam is near the liner, as it is on injection, a reasonable guess for d is the distance from the liner, so $d \sim 2\rho_b$. In this case the ion density can be enhanced by perhaps a factor of 2.

Thus between the expulsion of plasma electrons from the beam region, and partial trapping of plasma ions within the beam, the neutralizing ion density within the beam should be at least as large as the initial ion density in the plasma. Finally we note that on the time scale of beam acceleration, there should be no plasma electrons in the system. Therefore the accelerating field will not be shielded out by any background plasma, and the beam should accelerate as would a vacuum betatron.

C. The Ion Resonance and Ion Streaming Instability

Another potential problem with the plasma assisted modified betatron is the ion resonance instability. This is a particular concern because it is now established that two other similar devices, HIPAC¹⁶ and SPAC II¹⁷ were disrupted by the ion resonance instability. However in both of these devices the beam nearly filled the chamber, making it particularly susceptible to the $\ell = 1$ instability. For the modified betatron with $\rho_b \ll a$, the $\ell = 1$ mode should not go unstable and the main danger is an $\ell = 2$ mode. This mode was not observed on HIPAC or SPAC II. In the modified betatron, even if parameters are right for it, there is still a good chance that it will be stabilized by the diffuse profile.¹⁵

The ion resonance instability can occur if the ion bounce frequency is roughly equal to the $\ell = 2$ diocotron mode frequency. According to Ref. 15, this can occur only if

$$\gamma < \left(\frac{\omega_{be}}{2\omega_{ce}} \right)^{1/2} \left(\frac{M}{2m} \right)^{1/4}. \quad (31)$$

For our standard parameters for 10 kA beam, $\omega_{be} \sim 2.4 \times 10^{10}$, $\omega_{ce} \sim 4 \times 10^{10}$, $M/m = 1800$, Eq. (31) above reduces to $\gamma < 3$. Thus as long as $\gamma > 3$ after self field diffusion, the plasma background should not give rise to an ion resonance instability.

Another concern is the ion streaming instability, also discussed in Ref. 15. In this instability a cyclotron mode on the beam resonates with the stationary ions. The instability only occurs if the parallel wave number is in the range

$$\frac{\omega_{ce}}{\gamma c} < k < \left(\left(\frac{\omega_{ce}}{\gamma c} \right)^2 + \frac{\omega_{pe}^2}{\gamma^2 c^2} \right)^{1/2} \quad (32)$$

and the growth rate is roughly

$$\text{Im}\omega \sim \frac{n_i}{n_e} \frac{\omega_{be}^2}{8\gamma^2 \omega_{ce} \left(\frac{\omega_{ce} \rho_b}{c}\right)^3}. \quad (33)$$

For our standard parameters, the growth time is about 20 μsec and the range of unstable k is several percent and is dependent on γ . The idea then is to accelerate the beam so that γ changes by several percent in a few growth times.

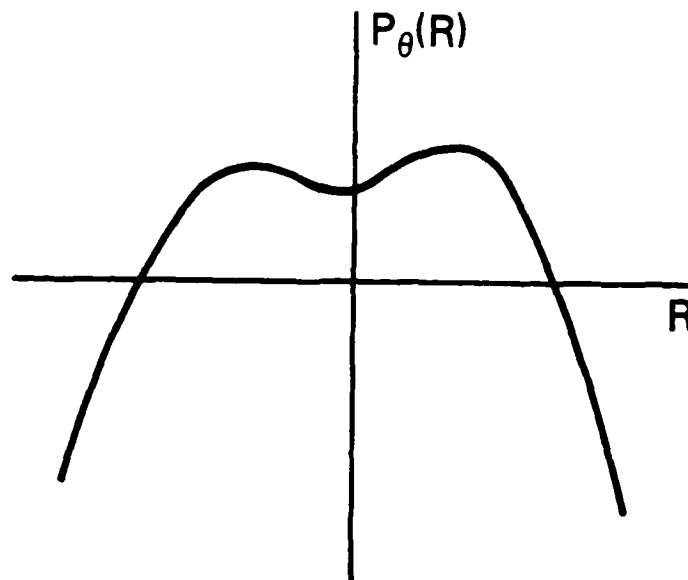
V. Conclusions

We have shown that a low density background plasma can significantly aid the modified betatron by allowing high current injection, avoiding the potential disruption on diamagnetic to paramagnetic transition and stabilizing the $l = 2$ resistive wall instability. There are difficulties regarding the plasma production and the ion resonance instability. However ways to overcome these problems do exist and appear to be feasible.

Acknowledgments

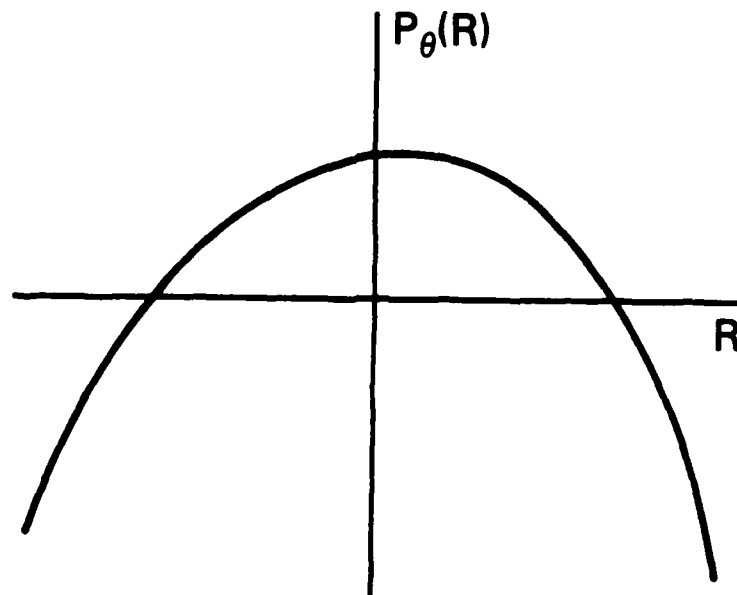
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DIAMAGNETIC BEAM



(a)

PARAMAGNETIC BEAM



(b)

Fig. 1 a) The dependence of P_θ on radius for a diamagnetic beam. At the reference orbit, P_θ is a relative minimum.
b) Same for a paramagnetic beam. At the reference orbit, P_θ is a relative maximum.

Appendix

In this appendix we discuss some of the recent simulations¹⁸ of a beam in the modified betatron as it is accelerated through the diamagnetic to paramagnetic transition. In Ref. 18, a 10 kA beam was accelerated from 3 MeV to 10 MeV by linearly ramping up the vertical field over a time of about 500 nsec. If the beam has no spread in longitudinal energy, it seems to be able to survive this transition with about a 30% increase in emittance and beam radius. On the other hand, with a 1% spread in longitudinal energy, the emittance increased by about an order of magnitude and the beam radius also increased significantly. In addition, for the latter case, the phase space plots clearly show particles scraped off the beam edge and drifting out along a separatrix.

The simulation, at least the first (constant longitudinal energy) case, is considerably more optimistic than what the fluid theory of Refs. 4-6 would predict. The difference in the two results probably has its origin in the difference in time scales. In both the fluid theory and the simulations, P_0 is an exact constant of the motion. However the fluid theory has one additional constant of motion, the toroidal flux through the drift orbit. The significant point is that this flux is not an exact constant, but an adiabatic invariant. Therefore for it to be conserved, the changes in the orbit have to be very slow compared to the poloidal drift orbit time. If the time for changes are comparable to a drift orbit time, the whole concept of an adiabatic fluid evolution of these drift orbits breaks down.

For the modified betatron, the drift orbit time is about 200 nsec or more. In the simulations, the energy went from 3 MeV to 10 MeV in 500 nsec, so it is extremely unlikely that an adiabatic theory would be valid. Since there is only one constant of motion, P_0 , the transverse orbits would most

likely fill up the beam region ergodically. However in the modified betatron experiment, the acceleration time is milliseconds. Thus, at least up to transition, and shortly after transition, the adiabatic theory should be valid. Of course adiabatic theory cannot be valid at transition itself, as pointed out in Refs. 4-6, and it may be that even for the slow time scale acceleration, the beam may pull itself through transition much more quickly than the time scale indicated by change in vertical field. Results from the fast time scale simulations indicate that this is the case. However at least during the early phase of transition, the adiabatic theory does indicate that the outer part of the beam is scraped off onto a separatrix whose size, depending on the parameters, can be comparable to the liner radius.

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